2020
MATHEMATICS
[HONOURS]
Paper : VIII

Full Marks : 50 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **five** questions: 2×5=10

   a) Define absolute error and relative percentage error of a number.

   b) Define the degree of precision of a quadrature formula. What is the degree of precision of the Trapezoidal rule?

   c) Establish $\Delta = E.\nabla = \nabla.E$ where Δ-forward difference operator, E-shift operator and ∇-backward difference operator.

   d) Write down the condition of convergence of Newton-Raphson method for finding the real root of the equation f(x)=0.

   e) Write down the limitations of Taylor's series method in solving a first-order differential equation with given initial condition.

   f) Write down Fortran expression of the following $g = \dfrac{e^{-x^2}\sin^2(x+|x|)}{\cos(x+\sqrt{y})} + x\log_e y$.

   g) Explain with an example and show that the expressions $\dfrac{(I+J)}{K}$ and $\dfrac{I}{K}+\dfrac{J}{K}$ do not produce the same result.

   h) Find the binary equivalent of $(18)_{10}$ and $(13.05)_{10}$.

2. Answer any **three** questions: 8×3=24

   a) i) Establish Lagrange's interpolation formula. Mention one advantage and one disadvantage of using this formula.

   ii) Given f(0)=3, f(1)=12, f(2)=81, f(3)=200, f(4)=100 and f(5)=8. Find $\Delta^5 f(0)$. (4+2)+2

b) i) Deduce Simpson's one-third formula from Newton-Cote's quadrature formula for numerical integration of f(x) in [a, b]. Explain geometrically why this rule is called a parabolic type.

ii) If T_1 and T_2 denote the trapezoidal approximations to $I=\int_a^b f(x)dx$ with single interval and double subintervals respectively then show that $I = T_2 + \frac{1}{3}(T_2 - T_1)$. (3+2)+3

c) i) Explain the Newton-Raphson method to determine approximately one real root of the equation f(x)=0. Write down the iteration formula for finding the q-th root of a positive real number R.

ii) Write down the geometrical interpretation of Regula-Falsi method for finding a real root of the equation f(x)=0. (3+2)+3

d) i) Write down the Gauss-Seidel Iteration scheme for finding the solution of an n×n system of linear equations. When does it fail to determine the solution?

ii) Prove that:

$$\Delta^k f(x) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} f[x+(k-i)h]$$

where h is the step length and $\Delta^k f(x)$ is the k-th order forward difference operator. (3+1)+4

e) i) Explain Euler's method for solving first order differential equation with given initial condition.

ii) Using fourth order Runge-Kutta method, find y(1.1) and y(1.2) with h=0.1 for the differential equation $\frac{dy}{dx} = x^2 + y^2$, y(1)=0. 4+4

3. Answer any **two** questions: 8×2=16

a) i) State two differences between a compiler and an interpreter.

ii) Point out the error of the following statements

A+B=B+A and

Force=Mass∗Acceleration

iii) Draw a flow chart for finding the value of $\int_a^b f(x)\,dx$ by trapezoidal rule.

2+2+4

b) i) Write short notes on:

Logical IF and Assignment statements.

ii) Write a FORTRAN program to compute the sum of the series

$$1+x+\frac{x^2}{\underline{|2}}+\frac{x^3}{\underline{|3}}+\ldots+\frac{x^{10}}{\underline{|10}}$$

for a given value of x. (2+2)+4

c) i) Write an algorithm to find the solution of f(x)=0 by fixed point iteration method.

ii) Write a FORTRAN program to compute the value of $\left(x^2+1\right)\left(x^3+1\right)^{\frac{1}{3}}$ for x varying from 0 to 20 at steps 2.

4+4

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