U.G. 4th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT402

Course Title : Partial Differential Equation, Laplace Transform and Tensor Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) State Ricci Lemma.

b) Define 'dummy index' with example.

c) Prove that $\delta^i_k \delta^k_u \delta^u_i = n$.

d) Define inner product of two tensors.

e) Define orthogonality of two vectors for covariant components.

f) Prove that $\delta^i_{j,k} = 0$.

g) Define a quasi-linear partial differential equation.

h) Write down the Lagrange's auxilary equation for the partial differential equation $y^2p - xyq = x(z-2y)$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

i) What is the complete integral of p+2q=0?

j) Define degree of a partial differential equation.

k) What will be the degree of the partial differential equation $\frac{\partial z}{\partial x} = \frac{5z}{\frac{\partial z}{\partial x}}$.

l) Find the Laplace transform of $F(t) = \frac{e^{at}-1}{a}$.

m) Find $L^{-1}\left\{\frac{1}{2s-5}\right\}$.

n) State initial value theorem for Laplace transform.

o) If $L\{f(t)\} = \frac{e^{-\frac{2}{s}}}{s}$, find the value of $L\{f(3t)\}$.

2. Answer any **five** questions: 2×5=10

a) If A_{ij} is a skew symmetric tensor, show that $\left(\delta^i_j \delta^k_l + \delta^i_l \delta^k_j\right) A_{ik} = 0$.

b) If A_i is a tensor of type (0, 1), then show that $\frac{\partial A_i}{\partial x^k}$ is not in general a tensor. Write down the condition for which it becomes a tensor.

c) Prove that $[ij, k] = [ji], k$.

d) Find a partial differential equation by eliminating a and b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$.

e) Write down the Charpit's auxilary equations for the PDE $z^2(p^2z^2 + q^2) = 1$.

f) Show that the surfaces represented by $Pp + Qq = R$ are orthogonal to the surface represented by Pdx+Qdy+Rdz=0, where P, Q, R are functions of x, y and z.

g) If $L(F(t)) = f(s)$, then prove that $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)ds$, provided the integral exists.

h) Find $L^{-1}\left\{\frac{3s-8}{4s^2+25}\right\}$.

3. Answer any **two** questions: 5×2=10

a) Using Laplace Transformation method, solve the initial value problem $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}$, given that $y = -3$, $\frac{dy}{dt} = 5$ when t=0.

b) Find the complete integral of $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$.

c) Define Christoffel symbol of 1st and 2nd kind. Also, prove that $\left\{\begin{matrix} i \\ i\,j \end{matrix}\right\} = \frac{\partial}{\partial x^j}\log\sqrt{g}$, where $g = |g_{ij}| > 0$.

4. Answer any **one** question: 10×1=10

a) i) Prove that a skew-symmetric tensor of order two have at most $\frac{1}{2}n(n-1)$ different non-zero components in n-dimensional space.

ii) Prove the identities $g_{ij,k} = 0$, $g^{ij}_{jk} = 0$ and $\delta^i_{j,k} = 0$.

iii) If A^i and B_i are contravariant and covarriant vectors respectively, then show that A^iB_i is an invariant.

3+5+2=10

b) i) Solve by Charpit's method the PDE $(x^2 - y^2)pq - xy(p^2 - q^2) = 1$.

ii) State and prove quotient law for tensor of type (0, 2).

iii) Evaluate $L^{-1}\left\{e^{-4s}/(s-3)^4\right\}$. 5+3+2=10

c) i) Apply the convolution theorem to prove that

$$B(m, n) = \int_0^t u^{m-1}(1-u)^{n-1}\, du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},$$

$m > 0,\ n > 0$.

ii) If $A^k_{ij}B^{il}_k = 0$ for every B^{il}_k, prove that A^k_{ij} vanishes identically.

iii) Discuss the geometrical interpretation of complete integral for the PDE $f(x, y, z, p, q) = 0$. 5+3+2=10
