U.G. 6th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT601

Course Title : Numerical Methods & Computer Programming

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

   a) State the condition of convergence of Newton–Raphson method.

   b) Write one limitation of Taylor's series method in solving a first order differential equation with given condition.

   c) When is a matrix said to be diagonally dominant?

   d) Define degree of precision of an interpolating quadrature formula.

   e) State one reason why polynomial in choosen as an interpolating function instead of several other functions.

   f) What is meant by an executable statement?

   g) What are source and object programs?

   h) Is the C-statement 5+3=8 wrong? Justify.

   i) Convert $(2AB)_{16}=(?)_{10}$.

   j) Write syntax of if statement in C.

   k) Find the value of $\frac{5}{3}+\frac{3}{5}-6*\frac{3}{15}$.

   l) Prove that $\Delta.\nabla=\Delta-\nabla$ where the symbols Δ and ∇ have usual meanings.

   m) Determine the number of significant figures in 8.1205 given its absolute error as 0.3×10^{-2}.

   n) State the limitations of Simpson's 1/3rd rule.

   o) Calculate the sum $(110111.11)_2+(11011101.01)_2$.

2. Answer any **five** questions: 2×5=10

   a) State the sufficient condition for convergence of the Gauss-Seidel method for the solution of an n×n system of linear equations.

b) Given f(0)=–1, f(1)=1, f(2)=4, find $\int_0^2 f(x)dx$ by Trapezoidal Rule.

c) Establish the result $\Delta^n f_0 = \nabla^n f_n$, the symbols have their usual meanings.

d) Prove that $\Delta f(x) = \log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$ where Δ is the forward difference operator.

e) Write the equivalent C-expression for the mathematical expression

$$\log_e\left|\frac{x+\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\right|.$$

f) Write a short note on switch statement in C.

g) Rounding the number 0.16152 to three significant digits, determine the absolute error and relative error of the obtained approximate number.

h) Write down the hierarchy of arithmetic operators.

3. Answer any **two** questions: 5×2=10

a) Describe modified Euler's method for solving a differential equation of the form $\frac{dy}{dx} = f(x,y)$, with $y_0 = y(x_0)$.

b) i) Write a program in C that reads the values of x and y, computes and prints the value of F(x) defined by

$$F(x) = 1+\frac{x}{\sqrt{1+x^2}}, \text{ if } |x|<3$$
$$= 0, \quad \text{if } 3\le|x|<a$$
$$= 1-\frac{x}{\sqrt{1+x^2}}, \text{ if } |x|\ge a \text{ where } a=(4+|y|)^{\frac{3}{2}}.$$

ii) Find the rate of convergence of the iteration method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

3+2

c) i) Find the quadratic polynomial which takes the same values as f(x) at x= –1, 0, 1 and integrate it to prove that

$$\int_{-1}^{1} f(x)dx = \frac{1}{3}\left[f(-1)+4f(0)+f(1)\right].$$

Assuming the error to have the form $Af^{iv}(\xi), (-1<\xi<1)$, find the value of A.

ii) In a Boolean algebra B, for any $a, b \in B$, Prove that $a + a.b = a$. 4+1

4. Answer any **one** question: 10×1=10

a) i) Establish Newton's Forward interpolation formula in standard form. Why is it called 'forward'?

ii) Derive the error in Simpson's $\frac{1}{3}$rd rule from Newton Cote is quadrature formula. Explain geometrically, why this rule is called a parabolic rule. (4+1)+(4+1)

b) i) Describe Gauss' elimination method for the solution of a system of n linear equations with real coefficients in n unknowns.

ii) Write a program in C to find a real root of the equation $x^3 - 5x + 1 = 0$ by Newton-Raphson method correct up to 4 decimal places.

iii) Write short notes on 'do-while loop' in C. 4+4+2

c) i) Draw a flow chart with conventional symbols to integrate a function $f(x)$ in $[a, b]$ by Trapezoidal rule.

ii) Express the Boolean function $xyz + xy'z' + x'yz' + x'y'z'$ in conjunctive normal form.

iii) Use Runge-Kutta fourth order method to solve the equation

$$\frac{dy}{dx} = x + y, y(0) = 1 \text{ at } x = 0.1 \text{ and } x = 0.2$$

3+3+4

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