U.G. 1st Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCHT102

Course Title : Algebra-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $1\times10=10$

a) Express $\sqrt{5}$ as a simple continued fraction.

b) Apply Descartes' rule of signs to ascertain the minimum number of complex roots of the equation $x^7-3x^3+x^2=0$.

c) If α, β, γ be the roots of the cubic equation $x^3+px^2+qx+r=0$, find the value of $\sum\alpha^2\beta^2$.

d) Find the power set of $S=\{0,\theta,\{1,2\}\}$.

e) If $iz^2-\overline{z}=0$, find the value of $|z|$.

f) If the diophantine equation 231x+35y=11 solvable?

g) Find the number of common roots of the equations $x^{24}-1=0$ and $x^{36}-1=0$ in the set of complex numbers.

h) Let S be a finite set having 7 elements, find the number of reflexive relation defined on S.

i) Prove that for any integer n, $n(n+1)^2>4(n!)^{\frac{3}{n}}$.

j) If $a_1, a_2, ..., a_n$ be n positive real numbers in ascending order of Magnitude, prove that $\frac{a_1^2+a_2^2+...a_n^2}{a_1+a_2+...a_n}<a_n$.

k) Find the principal value of $(-i)^{-i}$.

l) Find the number of positive divisors of 2700.

m) If α and β are the roots of the equation $x^2+1=0$, find the value of $\alpha^{2021}+\beta^{2021}$.

n) Let r be an integer such that $1\le r\le n$ and let A be a set having r elements and B be a set having n elements. Write down the number of injective mappings defined from A to B.

o) Find the number of special roots of the $x^{64}-1=0$.

2. Answer any **five** questions: 2×5=10

a) Give an example of a relation on a set which is reflexive and transitive, but not symmetric.

b) If a, b, c be positive real numbers, prove that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3$.

c) If n be an odd positive integer, prove that $\phi(2n)=\phi(n)$.

d) If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, find the value of $|z_1+z_2+z_3|$.

e) Let A and B be two finite sets with $|A|=5$, $|B|=3$, find the number of surjections from A to B.

f) If $1, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ are the roots of the equation $\alpha^n-1=0$, find the value of $(1-\alpha_1)(1-\alpha_2)\ldots.(1-\alpha_{n-1})$.

g) Find the maximum value of $(x+2)^5(7-x)^4$, when $-2<x<7$.

h) If $f:A\to B$ and $g:B\to C$ be two mappings such that $gof:A\to C$ is surjective, prove that g is surjective.

3. Answer any **two** questions: 5×2=10

a) Find the equation of the squared differences of the roots of the cubic $x^3+x^2-x=1$. Hence show that two roots of this equations are equal. 4+1

b) i) Using principle of induction, prove that $3^{2^n}-1$ is divisible by 2^{n+1} for all $n\in\mathbb{N}$.

ii) Prove that the set of all positive divisors of 12 under divisible relation form a lattice. 3+2=5

c) i) State Descartes rule of sign.

ii) Let $f:S\to\mathbb{R}$ defined by $f(x)=\frac{x}{1-|x|}, x\in S$ where $S=\{x\in\mathbb{R}:-1<x<1\}$.

Show that f is a bijection and also determine f^{-1}. 1+4=5

4. Answer any **one** question: 10×1=10

a) i) Solve by Ferrari's method $x^4+12x-5=0$.

ii) If z is a variable complex number such that an amplitude of $\frac{z-i}{z+i}$ is $\frac{\pi}{4}$. Show that the point z lines on a circle in the complex plane.

iii) Prove that the equation $(x+1)^4=a(x^4+1)$ is a reciprocal equation if $a\neq 1$. 5+3+2=10

b) i) State and prove Fermat's theorem.

ii) If $\cos^{-1}(u+iv)=p+iq$, where p, q, u, v are real, prove that $\cos^2 p$ and $\cosh^2 q$ are the roots of the equation $x^2-(1+u^2+v^2)x+u^2=0$.

iii) Find the special root of the equation $x^{20}-1=0$. Deduce that $\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{7\pi}{10}, \cos\frac{9\pi}{10}$ are the roots of the equation $16t^4-20t^2+5=0$.

3+2+5=10

c) i) Prove that an equivalence relation *P* on a set S determines a partition of S and conversely, each partition of S yields an equivalence relation on S.

ii) Discuss Cardan's method of solution of the cubic $Z^3+3Hz+G=0$, where suppose that H and G to be real and that $G^2+4H^3>0$.

iii) Find the Hasse diagram of the poset $(S, \leq)$, where $S=\{1,2,3,4,6,12\}$ and $a\leq b$ means "b is divisible by a".

(2+3)+3+2=10

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