U.G. 2nd Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT202

Course Title : Ordinary Differential Equations and Linear Algebra

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Define basis of a vector space.

b) Show that the functions 1 and e^{-x} are linearly independent solutions of $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ in the interval $(-\infty,\infty)$.

c) Give geometrical interpretation of Pdx+Qdy+Rdz=0, where P, Q, R are functions of x, y, z.

d) What is Bernoulli's differential equation?

e) Write down the conditions for consistency of the system of n equations AX=B with n - unknown.

[Turn Over]

f) Solve $x^2\frac{dy}{dx}+xy+\sqrt{1-x^2y^2}=0$

g) Let S and T be two subsets of vector space V over a field F, then show that $S\subseteq T\Rightarrow L(S)\subseteq L(T)$.

h) Define trajectory of a given family of curves.

i) Show that $W=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2=5\}$ is not a subspace of $\mathbb{R}^3$.

j) Give an example of a linearly independent set.

k) When are two systems AX = B and CX = D called equivalent systems?

l) Transform the differential equation $3x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+2y=0$ to a differential equation with constant coefficient.

m) Defined row-reduced echelon matrix.

n) Check the integrability of $(y^2+yz)dx+(z^2+zx)dy+(y^2-xy)dz=0$

o) If $\alpha,\beta\in\mathbb{R}^2(\mathbb{R})$ then show that the set $\{\alpha,\beta,a\alpha+b\beta\}$ where $a,b\in\mathbb{R}$ is dependent.

2. Answer **five** questions: $2\times5=10$

a) Find the differential equation of all circles touching the y axis at the origin.

b) Let W_1 and W_2 be two subspaces of a vector space V (F). Prove that $W_1+W_2=\{w_1+w_2 : w_1 \in w_2 \text{ and } w_2 \in w_2\}$ is a subspace of V(F).

c) Obtain a basis of $\mathbb{R}^3$. containing the vectors 2, –1, 0) and (1, 3, 2).

d) Prove that the number of integrating factors of an equation Mdx + Ndy = 0, which has a solution, is infinite.

e) Find the coordinate vector of $\alpha=(1,3,1)$ relative to the ordered basis $B=\{\alpha_1,\alpha_2,\alpha_3\}$ of $\mathbb{R}^3$, where $\alpha_1=(1,1,1)$, $\alpha_2=(1,1,0)$, $\alpha_3=(1,0,0)$.

f) Find the particular integral of the differential equation $\frac{d^2y}{dx^2}+y=\sum_{k=0}^{\infty}\frac{3.(4x)^k}{k!}$.

g) Solve $\frac{d^2x}{dt^2}-3x-4y=0, \frac{d^2y}{dt^2}+x+y=0$.

h) Find the singular solution of the differential equation

$$\sin\left(x\frac{dy}{dx}\right)\cos y=\cos\left(x\frac{dy}{dx}\right)\sin y+\frac{dy}{dx}.$$

3. Answer any **two** questions: $5\times2=10$

a) Show that a necessary and sufficient condition for a non-homogeneous system AX = B to be consistent is rank of A = rank of $\bar{A}$, where $\bar{A}$ is the augmented matrix of the system.

b) Solve

$$(y^2+yz+z^2)dx+(z^2+zx+x^2)dy+(x^2+xy+y^2)dz=0$$

c) i) Determine the row rank of the matrix A, where

$$A=\begin{pmatrix}2&1&4&3\\3&2&6&9\\1&1&2&6\end{pmatrix}$$ 3

ii) Show that the general solution of the equation $\frac{dy}{dx}+Py=Q$ can be written in the form y = k (u – v) + v, where k is a constant and u and v are its two particular solutions. 2

4. Answer any **one** question: 10×1=10

a) i) Solve $(D^2-1)y=50xe^{2x}\cos x$.

ii) If $y=\phi(x)$ be a known non - trivial solution of the 2nd order linear homogeneous differential equation $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ on some interval I, then $y=\phi(x)\int\frac{e^{-\int P(x)dx}}{[\phi(x)]^2}dx$ is another solution of it on I and these two solutions are linearly independent on I. 6+4

b) i) If a non-null vector space V over a field F be spanned by a linearly dependent set $\{\alpha_1,\alpha_2,...,\alpha_n\}$, then prove that V can also be spanned by a suitable proper subset of S.

ii) Extend the set {(1,1,1,1), (1,-1,1,-1)} to a basis of $\mathbb{R}^4$.

iii) Stating with the augmented matrix, find the value of K for which the system of equations x+y+z = 2, 2x+y+3z = 1, x+3y+2z = 5, 3x-2y+z = k is solvable and then solve it. 3+4+3

c) i) Solve, by the method of variation of parameters, the equation
$$\frac{d^2y}{dx^2}+a^2y=\sec ax.$$

ii) Show that the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal. 6+4

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