U.G. 6th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMDSHT4

Course Title : Probability and Statistics

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Give the classical definition of probability.

b) A coin is tossed two times in succession. Find the probability of getting one head.

c) Define distribution function of a random variable X.

d) Define probability mass function for a discrete random variable X.

e) Prove that $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$; a, b are constants.

f) **True** or **false** : Kurtosis means deviation from symmetry.

g) Define conditional expectations in bivariate distribution.

h) If two regression lines are mutually perpendicular then what will be the correlation co-efficient?

i) Find the moment generating function of Poisson distribution.

j) Define conditional variance of X, given Y=y in terms of expectation.

k) Define scatter diagram.

l) For Poisson distribution if its probability mass function is f_i show that $\sum_i f_i = 1$.

m) Write down the density function of Student's t-distribution and mention its parameter.

n) When a statistic T is said to be an unbiased estimator of a parameter θ?

o) What do you mean by level of significance in testing of hypothesis?

2. Answer any **five** questions: $2\times5=10$

a) Show that the probability that exactly one of the events A and B occur is $P(A)+P(B)-2P(AB)$.

b) Prove that $\mathrm{Cov}(X, Y)=E(XY)-E(X)E(Y)$.

c) Show that the mean deviation about the mean of a normal (m, σ) distribution is $\sqrt{\frac{2}{\pi}}\sigma$.

d) The random variables X, Y are connected by the linear relation 2X+3Y+4=0. Show that $\rho(X, Y)=-1$.

e) Find the first four central moments for the set of numbers 1, 3, 6, 7, 8.

f) If X is a binomial (n, p) variable, p being unknown, find an unbiased estimator of $p^2(n>1)$.

g) Find the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha-x)$, $0<x<\alpha$ for a sample of unit size.

h) Distinguish between statistic and parameter.

3. Answer any **two** questions: $5\times2=10$

a) Let a random variable X follow the normal (m, σ) distribution. Write down its probability density function f(x) and show that $\int_{-\infty}^{\infty} f(x)\,dx=1$. 1+4=5

b) State and prove Tchebycheff's Inequality. 5

c) The joint probability density function of the random variable X and Y is

$$f(x, y) = k(3x+y), \quad \text{when } 1\le x\le 3,\ 0\le y\le 2$$
$$= 0, \quad \text{elsewhere}$$

Find:

i) the value of k

ii) $P(X+Y<2)$

iii) the marginal density functions of X and Y. Investigate whether X and Y are independent. 5

4. Answer any **one** question: 10×1=10

a) i) The joint density function of X and Y is given by

$$f(x,\ y)=\begin{cases} K(x+y), & 0<x<1,\ 0<y<1 \\ 0, & \text{elsewhere}. \end{cases}$$

Find the value of K. Hence find the marginal density function $f_X(x)$ and the conditional density function $f_Y(y|x)$. 1+2+2=5

ii) Let X be a Poisson μ -variate. Determine Var(X). Find the moment generating function of a binomial (n, p) variate. 3+2=5

b) i) If the correlation co-efficient $\rho(X,\ Y)$ between two random variables X and Y exists, then prove that $-1 \le \rho(X,\ Y) \le 1$. 4

ii) Suppose we draw two observations X_1 and X_2 at random from $N(\mu,\ \sigma^2)$ and wish to estimate μ. We define an estimator of μ as $T = aX_1 + bX_2$. What values should we give to a and b so that T will be an unbiased estimator of μ and the minimum-variance unbiased estimator of μ? 6

c) i) The density function of a two-dimensional random variable (X, Y) is given by

$$f(x,\ y)=\begin{cases} 2, & \text{if } 0<x<y<1 \\ 0, & \text{elsewhere}. \end{cases}$$

Compute $P\left(X \ge \frac{1}{2} \middle| Y = \frac{2}{3}\right)$. 5

ii) If $X_1, X_2, ..., X_n$ be a random sample from a normal $(\mu,\ \sigma^2)$ distribution, then show that $\frac{(n-1)S^2}{\sigma^2}$ is a chi-square (χ^2) distribution with $(n-1)$ degrees of freedom, where S^2 is the sample variance. 5