U.G. 1st Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCHT102

Course Title : Algebra-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Express $-1+i$ in polar form.

b) Show that $\text{Log}(-1)=(2n+1)i\pi$.

c) Find the number of real roots of the equation $x^4+2x^2-7x-5=0$.

d) Find by synthetic division the quotient and the remainder when $3x^3+6x^2+5x-15$ is divided by $x+4$.

e) Define reciprocal equation.

f) Find the number of special roots of the equation $x^{72}-1=0$.

g) Use the principle of induction, prove that $1+3+5+...+(2n-1)=n^2$, for all $n\in N$.

h) Prove that $19^{20}\equiv1\pmod{181}$.

i) Give an example of antisymmetric relation defined on R.

j) Prove that

$$\left[A\cap(B\cup C)\right]\cap\left[A'\cup(B'\cap C')\right]=\phi.$$

k) Examine, whether the mapping $f:\mathbb{R}\to\mathbb{Z}$ defined by $f(x)=[x]$, $x\in\mathbb{R}$ is bijective?

l) Show that $a^2+b^2+c^2\geq ab+bc+ca$, where $a, b, c\in R$.

m) Find $\sigma(360)$.

n) Give an example of a partial order relation.

o) Find the relation between a and b, if (ax^5+3x^3b+8) be exactly divisible by $(x-2)$.

2. Answer any **five** questions: $2\times5=10$

a) Show that one of the values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}+\left(1-i\sqrt{3}\right)^{\frac{3}{4}}$ is $\sqrt[4]{32}$.

b) A relation ρ is defined on $\mathbb{R}$ by the rule $x\rho y$ if and only if $x-y$ is irrational. Examine whether ρ is transitive?

c) If x^2+px+1 be a factor of ax^3+bx+c, prove that $a^2-c^2=ab$.

d) Express $\frac{41}{29}$ as simple continued fraction.

e) If α be a special root of the equation $x^8-1=0$, prove that

$$(\alpha+2)(\alpha^2+2)\ldots(\alpha^7+2)=\frac{2^8-1}{3}.$$

f) If α, β, γ be the roots of the equation $x^3-2x^2+3x-5=0$, find the value of $\sum\frac{1}{\alpha\beta}$.

g) Show that $(n+1)^n > 2^n \underline{|n}$.

h) Find the general value of i^i.

3. Answer any **two** questions: 5×2=10

a) State the principle of induction for a set of positive integers. Using this principle, prove that $3^{2n}-8n-1$ is divisible by 64, where n is an arbitrary positive integer. 1+4

b) A relation ρ is defined on $\mathbb{Z}$ by "$a\rho b$ if and only if a^2-b^2 is divisible by 5 for $a, b \in \mathbb{Z}$". Prove that ρ is an equivalence relation on $\mathbb{Z}$. Also, find all the distinct equivalence classes.

c) i) Prove that $\tan\left(i\log\frac{a-ib}{a+ib}\right)=\frac{2ab}{a^2-b^2}$.

ii) If a, b, c be positive, then show that $(a+b+c)(a^2+b^2+c^2)\nless 9abc$. 3+2

4. Answer any **one** question: 10×1=10

a) i) If p is a prime and a is prime to p, prove that $a^{p^2-p}\equiv 1(\text{mod } p^2)$.

ii) If z_1, z_2 and a are complex numbers where $a\neq 0$, prove that in general $a^{z_1}.a^{z_2}\neq a^{z_1+z_2}$. When does the equality hold?

iii) Prove that the equation $(x+1)^4=a(x^4+1)$ is a reciprocal equation of $a\neq 1$ and solve it when $a=-2$. 3+(2+1)+4

b) i) Let A, B, C are subsets of a Universal set S, prove that

$$(A\setminus B)\times C=(A\times C)\setminus(B\times C).$$

ii) Let $S=\{x\in\mathbb{R}:-1<x<1\}$ and $f:\mathbb{R}\to S$ be defined by $f(x)=\frac{x}{1+|x|}$, $x\in\mathbb{R}$. Show that f is a bijection. Also determine f^{-1}.

iii) If α, β, γ be the roots of $x^3+qx+r=0$, find the equation whose roots are $(\alpha-\beta)^2$, $(\beta-\gamma)^2$, $(\gamma-\alpha)^2$. 3+3+4

c) i) Prove that the roots of the equation $\frac{1}{x+1}+\frac{1}{x+2}+\frac{1}{x+3}=\frac{1}{x}$ are all real.

ii) Solve the equation $x^4+12x-5=0$ by Ferrari's method.

iii) If a_1, a_2, ..., a_n are all positive real numbers less than 1 and $s_n=a_1+a_2+\ldots+a_n$, then

$$1-s_n<(1-a_1)(1-a_2)\ldots(1-a_n)<\frac{1}{1+s_n}$$ and

$$1+s_n<(1+a_1)(1+a_2)\ldots(1+a_n)<\frac{1}{1-s_n}$$

provided in the last inequality it is assumed that $s_n<1$. 3+3+(2+2)

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