U.G. 3rd Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCHT302

Course Title : Algebra-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Define order of an element a in group G.

b) Find the order of the permutation $\alpha = (1736)(254)$ in the permutation group S_7.

c) Give an example of a non-commutative ring with unity.

d) Find all generators of the cyclic group $(\mathbb{Z}_8, +)$.

e) Suppose G is a group of order 20. Does G contain a subgroup of order 8?

f) In a group G, find the number of elements $a \in G$ such that $a^2=a$.

g) Show that the ring $M_2(\mathbb{R})$ is not an integral domain.

h) What is the characteristic of a field with 16 elements?

i) Let $G=\langle a\rangle$ be a cyclic group of order 35. Find $[G:\langle a^7\rangle]$.

j) Suppose (G, o) be a group and H be a non-empty subset of G. Write down a sufficient condition for H to be a subgroup of (G, o).

k) Let G be a group and $a \in G$. If $o(a)=20$ then find $o(a^3)$.

l) Does the ring $(\mathbb{Z}_5, +, \cdot)$ contain any divisor of zero? Give reason.

m) Give an example of an infinite integral domain which is not a field.

n) Is the subgroup $H = \{e, (12), (34), (12)(34)\}$ of S_4 cyclic? Give reason.

o) Give an example of a ring which contains elements a, b such that $(a+b)^2 \neq a^2 + 2ab + b^2$.

2. Answer any **five** questions: 2×5=10

a) Prove that every cyclic group is commutative.

b) Let G be a group and $a \in G$ be such that $o(a) = n$. If $a^m=e_G$ then show that n divides m.

c) If a be a unit in a ring R with unity, then prove that a is not divisor of zero.

d) Find out two elements of order 2 in the dihedral group D_3 of order 6.

e) Prove that the set $S = \{(a, 3b) : a, b \in \mathbb{Z}\}$ is a subring of the ring $\mathbb{Z} \times \mathbb{Z}$.

f) Find all distinct left cosets of $(6\mathbb{Z},+)$ in the group $(\mathbb{Z},+)$.

g) If G is a commutative group then prove that $H=\{a^2 : a \in G\}$ is a subgroup of G.

h) Prove that every group of prime order is cyclic.

3. Answer any **two** questions: 5×2=10

a) i) Let H be a subgroup of a group G and $a, b \in G$. Prove that aH=bH if and only if $a^{-1}b \in H$.

ii) Prove that every proper subgroup of a group of order 6 is cyclic. 3+2

b) i) Define center of a ring. Prove that the center of a ring R is a subring of R.

ii) Prove that every field is an integral domain. (1+2)+2

c) i) If R be a ring with unity 1, then prove that the characteristic of R be n if and only if n.1=0, where n is the smallest positive integer.

ii) Show by an example that every subgroup of a group G is cyclic but G is not cyclic. 3+2

4. Answer any **one** question: 10×1=10

a) i) State and prove Lagrange's theorem.

ii) Let H be a subgroup of a group G. Prove that for any $g \in G$, $gHg^{-1}=\{ghg^{-1} \mid h \in H\}$ is a subgroup of G and $|ghg^{-1}|=|H|$.

(1+4)+(3+2)

b) i) Prove that a cyclic group of order n has one and only one subgroup of order d for every positive divisor d of n.

ii) State and prove Fermat's theorem.

iii) Examine if the ring of matrices $\left\{\begin{pmatrix} a & b \\ 3b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ contains divisors of zero. 4+(1+3)+2

c) i) In the ring $(\mathbb{Z}_n,+,.)$, prove that $\bar{m}$ is a unit if and only if gcd (m, n)=1.

ii) Prove that the characteristic of a finite field is a prime number.

iii) Find the elements in $\mathbb{Z}_{12}$ which are zero divisors. 5+3+2

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