U.G. 6th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMDSHT6

Course Title : Point Set Topology

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Define a countable set.

b) State Schroeder-Bernstein theorem.

c) State Zorns lemma.

d) Define the subspace topology.

e) Is it possible to construct a topology on every set?

f) Give an example of open set in $\mathbb{R}$ with usual topology, which is not an open interval.

g) Let τ be a cofinite topology on $\mathbb{N}$. Then write any three element of τ.

h) Let $(\mathbb{Z}, \tau)$ be a cofinite topological spaces. Is $B=\{1, 2, 3, ..., 100\}$ closed in τ?

i) Write the closure of the set $S=\left\{1+\frac{1}{n} : n\in\mathbb{N}\right\}$ in usual topology on $\mathbb{R}$.

j) Define component of a topological space X.

k) Give an example of path component which is not closed.

l) State Baire category theorem.

m) Give the definition of a quotient topology.

n) State when a topological space is said to be compact.

o) State Ascoli-Arzela theorem.

2. Answer any **five** questions: $2\times5=10$

a) Show that if $\mathcal{A}$ is closed in Y and Y is closed in X, then $\mathcal{A}$ is closed in X.

b) Define basis for a topology on X.

c) Show that if a topological space is T_1 then every point set of the space is closed.

d) Either prove or disprove: Real numbers with co-countable topology is T_2.

e) Let X and Y be topological spaces. Give the definition of the product topology on $X \times Y$.

f) What do you mean by locally compact space?

g) Show that the components of open subsets of the real line are open intervals.

h) Prove that the interval $(0, 1]$ is not compact.

3. Answer any **two** questions: $5 \times 2 = 10$

a) Let A be a subset of a topological space X. Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A. 5

b) Define a Hausdorff space. Show that every finite set in a Hausdorff space is closed. 1+4

c) Prove that a space (X, τ) is connected if and only if no continuous function on X into the discrete two point space $\{0, 1\}$ is surjective.

4. a) i) Prove that a countable union of countable sets is countable.

ii) In a topological space (X, τ), show that a non-empty set $U (\subseteq X)$ is open in X if and only if it is the neighbourhood of each of its points. 5+5

b) i) Prove that a function f from a topological space X to a topological space Y is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A of X.

ii) Either prove or disprove: $[0, 1) \cup (1, 2)$ is homeomorphic to $(-\infty, -3) \cup [0, \infty)$. 5+5

c) i) Define path connected space.

ii) Show that every path connected space is connected but the converse may not be true.

iii) Show that closure of a connected subspace of a topological space is also connected. 2+4+4

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