U.G. 2nd Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT202

Course Title : Ordinary Differential Equations and Linear Algebra

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Write down the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} + 2.\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^5$.

b) What is the number of arbitrary constants in the complete primitive of the differential equation $\phi\left(x, y, \frac{dy}{dx}, \frac{d^3y}{dx^3}\right) = 0$.

c) Let F be a finite field with four elements. What is the total number of non-zero proper subspaces of the vector space F^2 over F?

d) Find the value of m which makes the differential equation

$$\left(a^2 - mxy - y^2\right)dx - (x+y)^2\,dy = 0$$

exact.

e) Give the geometrical interpretation of the differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$.

f) Interpret the simultaneous equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ geometrically.

g) Determine the most general function $F(x, y)$ such that the differential equation $F(x, y)dx + (1 - xy)xdy = 0$ is exact.

h) Find the Wronskian of the functions e^{-x}, $\sin x$.

i) Obtain a particular integral of $\left(D^2 + 1\right)y = \sin x$.

j) What is the linear span of an empty set of vectors?

k) Obtain a basis of $\mathbb{R}^3$ containing the vector $(-1, 0, 2)$.

l) Solve $x\frac{dy}{dx}+y=xy^2$.

m) Write down the general solution to the equation $x+2y+4z=0$; $(x, y, z)\in\mathbb{R}^3$.

n) Show that the set

$$V=\{(x, y, z)\in\mathbb{R}^3 : x+y+z=1\}$$

is not a subspace of $\mathbb{R}^3$.

o) Give an example of an infinite dimensional vector space.

2. Answer any **five** questions: 2×5=10

a) Obtain the complete primitive and the singular solution of $y=px+\sqrt{1+p^2}$.

b) Evaluate $\frac{1}{D^2-1}xe^x$, where $D\equiv\frac{d}{dx}$.

c) Find the differential equation of all parabolas of latus rectum $4a$ and axis parallel to y axis.

d) Show that the functions $\cos x$, $\cos^3 x$, $\cos 3x$ are linearly independent on $(-\infty, \infty)$.

e) Determine and integrating factor of $xydx-(x^2+2y^2)dy=0$ and hence solve the equation.

f) In $\mathbb{R}^3$, let $S=\{\alpha, \beta, \gamma\}$ and $T=\{\alpha, \alpha+\beta, \alpha+\beta+\gamma\}$. Show that $L(S)=L(T)$.

g) Examine whether or not $S=\{(x, y, z)\in\mathbb{R}^3 : x+y+z=0\}$ is a subspace of $\mathbb{R}^3$.

h) Find the coordinate vector of α in $\mathbb{R}^3$ relative to the basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha=(0, 3, 1)$, $\alpha_1=(1, 1, 0)$, $\alpha_2=(1, 0, 1)$, $\alpha_3=(0, 1, 1)$.

3. Answer any **two** questions: 5×2=10

a) i) Prove that a linearly independent set of vectors in a finite dimensional vector space V over a field F is either a basis of V or it can be extended to a basis of V. 3

ii) Let $S=\{\alpha_1, \alpha_2, \alpha_3\}$ with $\alpha_1=(1, 2, 0)$, $\alpha_2=(3, -1, 1)$ and $\alpha_3=(4, 1, 1)$. Find a proper subset of S that can generate $L(S)$. 2

b) Show that the orthogonal trajectories of the system of co-axial circles $x^2+y^2+2\lambda x+c=0$ form another system of co-axial circles $x^2+y^2+2\mu y-c=0$, where $\lambda,\ \mu$ are parameters, and c is a given constant. 5

c) i) Solve $\frac{d}{dx}\left(\cos^2 x\frac{dy}{dx}\right)+y\cos^2 x=0$ by reducing to normal form. 3

ii) Solve $9x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0$. 2

4. Answer any **one** question: 10×1=10

a) i) If $y=\phi_1(x)$ and $y=\phi_2(x)$ are any two solutions of $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$, in which P and Q are continuous functions on some interval I, then their Wronskian $W(\phi_1,\ \phi_2)$ is either identically zero or nowhere zero on I.

ii) Prove that the homogeneous system $AX=O$ containing n equations in n unknowns has a non-zero solution if and only if rank of $A < n$.

iii) Prove, by an example, that the union of two subspaces of V is not in general, a subspace of V. 4+4+2

b) i) Find the orthogonal trajectories of the family of cardioides $r=a(1-cos\,\theta)$, a being a variable parameter. 3

ii) Solve, by the method of variation of parameter, the equation $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$. 5

iii) By Picard's theorem, find first and second approximations of $\frac{dy}{dx}=xy$ with $y(0)=1$. 2

c) i) Solve the simultaneous equations $\frac{d^2x}{dt^2}+\frac{dy}{dt}+x+y=t$, $\frac{dy}{dt}+2x+y=0$; given that $x=y=0$ at $t=0$.

ii) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S=\{(x,\ y,\ z)\in\mathbb{R}^3: 2x+y-z=0\}$. Also find a basis of S. 5+5

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