U.G. 2nd Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT201

Course Title : Real Analysis-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) If $a\in\mathbb{R}$ and $0\le a<\varepsilon$ holds for every positive ε, show that a=0.

b) Choose the correct:

The set of all irrational numbers is

i) countably finite

ii) countably infinite

iii) uncountable

c) Write down the Sup A, where $A=\{x\in\mathbb{R}|x^2<1\}$.

d) State the Infimum (Completeness) property of $\mathbb{R}$.

[Turn Over]

e) Let $S\subseteq\mathbb{R}$. Define an isolated point of S.

f) Let A be a closed set and B be an open set. Show that A–B is closed set.

g) Define a Cauchy sequence of real numbers.

h) Find $\lim\limits_{n\to\infty}\frac{(n+1)^{2n}}{(n^2+1)^n}$.

i) Give an example of a sequence of rational numbers that converges to a irrational number.

j) If $a_n=(-1)^n$ then $\lim\limits_{n\to\infty}|a_n|=?$

k) Give an example of divergent sequences (x_n) and (y_n) such that the sequence (x_n+y_n) is convergent.

l) Find the value of the series $\sum\limits_{n=0}^{\infty}2^{-n}$.

m) State Cauchy's root test for a series $\sum\limits_{n=1}^{\infty}a_n$.

n) Does the series $\sum\limits_{n=1}^{\infty}\frac{2^n}{1+2^n}$ converge to a finite value? Justify.

o) Test the absolute convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\sin\frac{1}{n}$.

2. Answer any **five** questions: 2×5=10

a) State the Archimedean property of $\mathbb{R}$.

b) Prove that a finite subset of R is a closed set.

c) State Monotone Convergence Theorem for a sequence of real numbers.

d) Find the derived set of the set $\left\{\frac{1+(-1)^n}{n},\ n\in\mathbb{N}\right\}$.

e) Give an example of a convergent sequence of positive numbers which converges to 0.

f) Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n^2-1}}{n^2+1}$.

g) If $\sum_{n=1}^{\infty}a_n$ be a convergent series of positive real numbers, then determine whether $\sum_{n=1}^{\infty}a_n^2$ is convergent or not.

h) Test the convergence of the series

$$1+\frac{1}{1!}+\frac{2^2}{2!}+\frac{3^3}{3!}+\ldots,$$

by D'Alembert ratio test.

3. Answer any **two** questions: 5×2=10

a) Prove that the series

$$1+\frac{3}{7}x+\frac{3}{7}.\frac{6}{10}x^2+\frac{3}{7}.\frac{6}{10}.\frac{9}{13}x^3+\ldots$$

is convergent if $x\le 1$ and divergent if $x>1$. 5

b) State and prove the Sandwich Theorem for sequences of real numbers. 2+3=5

c) Test the convergence of the series

$$1+\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^5}+\frac{1}{2^4}+\ldots.$$

What can you say about the convergence of the above series by D'Alembert ratio test? 3+2=5

4. Answer any **one** question: 10×1=10

a) i) Prove that the set of all open intervals having rational end points is countably infinite. 5

ii) Prove that an upper bound u of a non-empty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\varepsilon>0$ there exists an element s in S such that $u-\varepsilon<s\le u$. 5

b) i) Let G be an open set in $\mathbb{R}$. Then prove that the complement of G in $\mathbb{R}$ is a closed set in $\mathbb{R}$. 4

ii) If a sequence (x_n) converges to a finite value l then prove that every sub-sequences of (x_n) also converges to l. 4

iii) Let $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, $n \geq 1$. Then lim sup x_n=?, lim inf x_n=? 1+1=2

c) i) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, $p > 0$. 5

ii) Define an absolutely convergent series of real numbers. Is every absolutely convergent series of real numbers convergent? Does the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ converge absolutely for a fixed value of x? 2+1+2=5

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