U.G. 4th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : BMTMCCHT401

Course Title : Dynamics of Particle

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) If v be the velocity and x, the distance traversed by a particle in time t, then show that its acceleration is $v\frac{dv}{dx}$.

b) If a particle moves in a straight line according to the law $x = a\sin(\mu t + b)$, prove that its velocity is given by $v^2 = (a^2 - x^2)\mu^2$.

c) If the velocity of a moving particle along x-axis is $\sqrt{1-x^2}$, prove that its motion is simple harmonic with time period 2π.

d) An impulse I changes the speed of a particle of mass m from v_1 to v_2. Show that the kinetic energy gained is $\frac{I}{2}(v_1 + v_2)$.

e) Under the influence of a force field $\vec{F}$, a particle of mass m moves along the ellipse $\vec{r} = a\cos\omega t\,\hat{i} + b\sin\omega t\,\hat{j}$. Prove that $\vec{r}\times\vec{F} = \hat{O}$ (the null vector).

f) Prove that time rate of change of K.E. of a particle is equal to its power.

g) A particle describes an equiangular spiral $r = ae^{\theta\cot\alpha}$ during its motion with constant angular velocity under a force P to the pole. Find the law of force.

h) Show that the motion along a smooth vertical cycloidal path is simple harmonic motion.

i) State the principle of conservation of linear momentum.

j) Show that the linear velocity of a particle moving in a central orbit varies inversely as the perpendicular drawn from the centre of force upon the tangent to the path.

k) The law of motion for a particle moving along a straight line is given by $2x = vt + v$. Show that the particle moves with uniform velocity.

l) Prove that the rate of change of kinetic energy of a particle is equal to its power.

m) Two equal elastic spheres each of mass m move in the same direction with equal velocities u and collide directly. Show that they interchange their velocities.

n) A body of mass M is moving with velocity V. After an explosion it is separated into two bodies of masses m_1 and m_2 moving with velocities v_1 and v_2 respectively. Find total energy of the explosion.

o) If the tangential and normal accelerations of a particle moving in a plane curve are equal in magnitude, find the expression for velocity.

2. Answer any **five** questions: $2\times5=10$

a) In a S.H.M., the distances of a particle from the middle point of its path at three consecutive seconds are a, b, c. Show that the time of complete oscillation is $\dfrac{2\pi}{\cos^{-1}\left(\dfrac{a+c}{2b}\right)}$.

b) A particle moving in a straight line starts from rest and acceleration at any time t is $a\left(1-t^2\right)$,

where 'a' is a constant. Show that the maximum velocity attained by the particle is $\frac{2}{3}a$.

c) Deduce from Kepler's law that the planet has only radial acceleration towards the sun.

d) Two perfectly elastic bodies of equal mass moving in the opposite directions with velocities v_1 and v_2 respectively collide directly; find the velocities after collision.

e) Show that power at any instant can be defined as the product of the tangential component of a force along the path and the velocity at that instant.

f) If a particle moves in a straight line with uniform acceleration, then show that the change in K.E. is equal to the work done by the acting force.

g) A particle moves in a force field $\vec{F}=\phi(r)\vec{r}$ where $\phi(r)$ is a scalar function. Prove that the angular momentum of the particle about the origin is constant.

h) If a particle moves with constant angular velocity about a point O in its plane of motion, show that cross-radial acceleration is proportional to its radial velocity, O being the origin.

3. Answer any **two** questions: $5\times2=10$

a) A ball is thrown from a point on a smooth horizontal ground with velocity v at an angle α to the horizon. Show that the total time for which the ball rebounds on the ground is $\frac{2v\sin\alpha}{g(1-e)}$, where e is the coefficient of restitution of the ball to the ground.

b) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3}{2}\frac{(u+v)x}{u^2+uv+v^2}$.

c) A particle describes a path which is nearly a circle about a centre of attractive force $\left(=\frac{\mu}{r^n}\right)$. Show that the orbit is stable if $n<3$.

4. Answer any **one** question: $10\times1=10$

a) i) A sphere impinges directly on an equal sphere at rest; show that a fraction $\frac{1}{2}\left(1-e^2\right)$ of the original kinetic energy is lost due to the impact.

ii) A particle moves with central acceleration $\mu\left(r+\frac{a^4}{r^3}\right)$, being projected from an apse at a distance 'a' with a velocity $2\sqrt{\mu}\,a$. Prove that it describes the curve $r^2\left(2+\cos\sqrt{3}\,\theta\right)=3a^2$.

4+6=10

b) i) A particle moves towards a centre of force, the acceleration at a distance x, being given by $\mu^2\left(x+\frac{a^4}{x^3}\right)$. If it starts from rest at a distance 'a', show that it will arrive at the centre in time $\frac{\pi}{4\mu}$.

ii) A particle under a central acceleration $\frac{\mu}{r^7}$ is projected from a point at a distance a with a velocity from infinity, the direction of projection making an angle 45° with the initial line. Show that the equation of the path is $r^2 = a^2(\cos 2\theta - \sin 2\theta)$. 4+6

c) i) Two perfectly inelastic bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy due to impact is $\frac{1}{2}.\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2$.

ii) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greater when the radius vector to the planet is at right angle to the major axis of the path and that it then is $\frac{2\pi ae}{T\sqrt{1-e^2}}$, where 2a is the major axis, e the eccentricity and T the periodic time. 4+6

———————