U.G. 6th Semester Examination - 2022

MATHEMATICS

Course Code : BMTMDSHT6 [DSE-6]

Course Title : Point Set Topology

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) "The set of all rational points whose coordinates are both rational in the coordinate plane $\mathbb{R}^2$ is countable". Is it true?

b) State Baire's category theorem.

c) Define subspace topology.

d) Define a linearly ordered set.

e) Find the open sets for the topological space (X,τ) where $X=\{a,b\}$, $\tau=\{\varphi,\{a\},X\}$.

f) State continuum hypothesis.

g) Give an example of a bounded metric.

h) What is the interior of the set [2, 3] with respect to $\mathbb{R}$ with discrete topology?

i) Define homeomorphism between two topological spaces.

j) Give an example of a non-metrizable topological space.

k) Write down a base for the discrete topology on X.

l) Define regular topological space.

m) "Any discrete space with more than one point is disconnected." Is it true?

n) Which of the following is not a topological property:

i) Compactness

ii) Connectedness

iii) boundedness

iv) metrizability

o) Let A, B be compact subsets of a metric space X. Is $A\cup B$ comapct?

2. Answer any **five** questions: $2\times5=10$

a) Define Zorn's lemma.

b) Define 'Finer' and 'Coarser' topology.

c) Let $X=\{a, b, c\}$ and $\tau=\{\phi,\{a\},\{b\},\{c\},\{a,c\}\}$. Whether (X,τ) forms a topology or not?

d) Give an example of a sequence B_1, B_2, ... of closed sets in a topological space X whose union is not closed.

e) Write down a neighbourhood base at x in $\mathbb{R}^2$ with usual topology.

f) Define a map to show that the open interval (a, b) in $\mathbb{R}$ with usual topology is homeomorphic to (0, 1).

g) Prove that every finite set in a Hausdorff space X is closed.

h) Define seperable topological space.

3. Answer any **two** questions: $5\times2=10$

a) Prove that the union of countably many countable sets is countable.

b) i) Give a counter example to show that the property of being a Cauchy sequence is not topological.

ii) Let us consider the real function $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^2$. Is it an open map? Justify. $3+2=5$

c) Prove that the image of a connected space under a continuous map is connected.

4. Answer any **one** question: $10\times1=10$

a) i) Prove that the collection P of all polynomials $p(x)=a_0+a_1x+...+a_mx^m$; with integral co-efficients, $a_0, a_1, ..., a_m$ is denumerable.

ii) Prove that the intervals [0, 1], [0, 1] and [0, 1] have the same cardinality.

$6+4=10$

b) i) Let τ be the collection of subsets of $\mathbb{N}$ consisting of all subsets of the form

$G_m=\{m, m+1, m+2, ...\}, m\in\mathbb{N}$

Show that τ is a topology on $\mathbb{N}$.

ii) Show that every closed subspace of a compact space is compact. $5+5=10$

c) i) Prove that every compact subspace of a Hausdorff space is closed.

ii) Show that any infinite subset of a discrete topological space X is not compact.

$6+4=10$

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