U.G. 6th Semester Examination - 2022

MATHEMATICS

Course Code : BMTMDSHT5 [DSE-5]

Course Title : Mechanics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Can a force and a couple in the same plane produce equilibrium?

b) Write down the mathematical expression for the work done by a force $\vec{F}$ in moving a particle round a closed curve C.

c) What do you mean by 'constraints on a system'?

d) What is the value of resultant (R) if two equal forces P acting at a point of a body at an angle 120^{o}?

e) Define central axis of a system of forces.

f) Give an example of a non-homogeneous and compressible fluid.

g) "All the properties are true for the actual work will also be true in case of the virtual work" – Is the statement true?

h) What do you mean by a deformable body?

i) Define ideal fluid.

j) Write down the conditions of equilibrium of a system of non-coplanar forces.

k) What is meant by the stress component τ_{xy} at a point (x, y, z) is a continuous?

l) Write down the equation for a gas in an adiabatic temperature change.

m) If ρ be the mean density of the body and ρ' be the density of the fluid. What is the condition between ρ and ρ' such that the solid can not float?

n) What is the relation between depth of centre of pressure and centre of gravity of a plane area immersed in a heavy liquid under gravity?

o) Write down the equation that determines the pressure at any point in the fluid.

2. Answer any **five** questions: 2×5=10

a) State the principle of virtual works for a particle.

b) Distinguish between body forces and contact forces with examples.

c) Write down the pressure-density relation for a perfect gas in isothermal change of state.

d) State Archimedes' principle for a floating body.

e) State the necessary and sufficient condition of equilibrium of any system of coplanar forces acting on a body.

f) Write down the stress matrix at a point in an ideal fluid, explaining the symbols used.

g) Distinguish between vapour and gas.

h) Two parallel and equal forces are acting on a system with opposite directions. What will be the resultant?

3. Answer any **two** questions: 5×2=10

a) A telegraph wire, stretched between two poles at distance d meter apart, sags n meter in the middle. Prove that the tension at the ends is approximately, $w\left(\frac{d^2}{8n}+\frac{7}{6}n\right)$, where w is the weight of unit length of the unstretched wire.

b) A mass of fluid where density at (x, y, z) varies inversely as (x+y+z) is under the forces $(y+z)^2-x^2$, $(z+x)^2-y^2$, $(x+y)^2-z^2$. Show that fluid will be at rest. Also find the surface of equal pressure. 3+2

c) A conical vessel of height h and vertical angle 2α, contains water whose volume is one-half that of the cone, if the vessel and the contained water revolve with uniform angular velocity ω and no water overflows, show that ω must not be greater than $\sqrt{\frac{2g}{3h}}\cot\alpha$.

4. Answer any **one** question: 10×1=10

a) i) Two forces 2p and p act along the lines whose equations are $y=x\tan\alpha, z=c$; and $y=-x\tan\alpha, z=-c$, respectively. Find the equation of the central axis.

ii) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is stable. 5+5

b) i) A solid sphere rests inside a fixed rough hemispherical bowl of twice its radius. Show that however large a weight is attached to the highest point of the sphere, the equilibrium is stable.

ii) Show that the necessary and sufficient condition for equilibrium of a fluid under the action of external forces whose components along the co-ordinate axes are X, Y, Z acting at a point (x, y, z) is

$$X\left(\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}\right)+Y\left(\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}\right)+Z\left(\frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}\right)=0 .$$

c) i) If T be the absolute temperature at a height z and T_0 be its value at the sea level in the atmosphere having convective equlibrium of temperature satisfying the law $p = k\rho^{\gamma}$, prove that

$$\frac{T}{T_0}=1-\frac{\gamma-1}{\gamma}.\frac{rz}{H(r+z)} ,$$

where r is the radius of the earth and H the heights of the homogeneous atmosphere.

ii) Find the general cartesian equations of the equilibrium of a string under coplanar forces. 5+5

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