U.G. 6th Semester Examination - 2022

MATHEMATICS

Course Code : BMTMDSHT4 [DSE-4]

Course Title : Probability and Statistics

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) What is random experiment?

b) Write down two axioms of mathematical probability.

c) A die is thrown. Find the probability of getting 'even face'.

d) For the Geometric distribution, if f_i is the probability mass function, verify that $\sum f_i = 1$.

e) The distribution function of a random variable X is given by

$$f(x)=\begin{cases} 0, & x \le 0 \\ x, & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$$

Evaluate $\rho(0.2 < x \le 0.6)$.

f) Prove that $|E(X)| \le E(|X|)$.

g) Choose the correct option: If Var(X)=1, then $\mathrm{Var}(2X \pm 3)$ is equal to

i) 5 ii) 13

iii) 4 iv) 2

h) Find the characteristics function of Binomial distribution.

i) Write down the density function of χ^2 distribution mentioning the range of the variable.

j) Give an example of a discrete distribution which has the same mean and variance.

k) State a necessary and sufficient condition for two continuous random variables X and Y to be independent.

l) Define regression line of Y on X.

m) What do you mean by a 'sufficient statistic'?

[Turn Over]

n) What are the two sufficient conditions for an estimator T_n to be consistent estimator of θ?

o) Define power of the test in testing of hypothesis.

2. Answer any **five** questions: $2\times5=10$

a) If A and B be any two evenets then prove that

$$P(\bar{A}+B)=1-P(A)+P(AB)$$

where $\bar{A}$ denotes the complement of A.

b) If X is a Poisson variate with parameter μ and P(X=0)=P(X=1), then find μ and $P(X\geq 1)$.

c) Let X be a binomial (n, p) variate. Calculate the mean of X.

d) Find the correlation coefficient of (2X–3) and (X+2), where X is a random variable.

e) The random variable X is uniformly distributed in (0, 1). Find the distribution of $Y=-2\log_e X$.

f) Find the variance of the sample mean $\bar{X}$, given that $E(\bar{X})=\mu$.

g) Find the maximum likelihood estimate of θ in

$f(x)=\frac{1}{\theta}e^{-\frac{x}{\theta}};\ x>0,\ \theta>0$ on the basis of a random sample of size *n*.

h) Write a short note on Best critical region.

3. Answer any **two** questions: $5\times2=10$

a) If X be a standard normal variable, find the probability density function of Y, where $Y=\frac{1}{2}X^2$.

b) The density function of the two dimensional random variable (X, Y) is given by

$$f(x,y)=\begin{cases} c(2x+5y), & 0\leq x\leq 3,\ 2\leq y\leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Find:

i) the value of c

ii) the marginal density function of X and Y

iii) the conditional density function

$f_x(x/y)$ and $f_y(y/x)$

c) For five observation of pairs (x, y) of variables x and y, the following results are obtained:

$$\sum x=15,\ \sum y=25,\ \sum x^2=55,\ \sum y^2=135,\ \sum xy=83$$

Find the equation of the line of regression and estimate the values of x and y, if y =12; x=8.

4. Answer any **one** question: 10×1=10

a) i) Find the mean and variance of Normal distribution.

ii) The heights in inches of 8 students of a college chosen at random, are as follows:

62.2, 62.4, 63.1, 63.2, 65.5, 66.2, 66.3, 66.5

Compute 98% confidence interval for the mean of the population of heights of the students of the college, assuming it to be normal and find the length of the interval.

Given P(t>2.998)=0.01 for 7 degrees of freedom. 5+5

b) i) The joint density function of the random variable X, Y is given by

$$f(x, y) = x + y,\ 0 < x < 1,\ 0 < y < 1$$
$$= 0, \quad \text{elsewhere}$$

Find the distribution of XY.

ii) If $X_1, X_2, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$ population, show that the estimator $T = \frac{1}{n+1}\sum_{i=1}^{n} X_i$ is a biased but consistent for μ. Hence obtain the unbiased estimator for μ. 5+5

c) i) If $(x_1, x_2, ..., x_n)$ be any random sample of size n drawn from a normal (m, σ) population, then show that the sampling distribution of the sample mean $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is normal $\left(m, \frac{\sigma}{\sqrt{n}}\right)$.

ii) If T is an unbiased estimate of a population parameter θ, then show that T^2 is a biased estimate of θ^2. 5+5

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