U.G. 5th Semester Examination - 2021
MATHEMATICS
Course Code: BMTMDSHT3 [DSE 3]
Course Title: Theory of Equations

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Prove that x^2+x+1 is a factor of $x^{10}+x^5+1$.

b) Expand $f(x)=x^4-4x^3+3x^2+3x+7$ as a polynomial in x–1.

c) Determine the multiple roots of the equation $x^5+2x^4+2x^3+4x^2+x+2=0$.

d) Prove that the roots of the equation $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x$ are all real.

e) Show that for all real values of λ, the equation

$$(x+3)(x+1)(x-2)(x-4)+\lambda(x+2)(x-1)(x-3)=0$$

has all its roots real and simple.

f) Form a biquadratic equation with rational coefficients two of whose roots are $\sqrt{3}\pm 2$.

g) Solve the equation $x^5-1=0$.

h) Define special roots of the equation $x^n-1=0$.

i) If α, β, γ be the roots of the equation $x^3+2x^2+1=0$, find the equation whose roots are $\alpha=\frac{1}{\alpha}, \beta+\frac{1}{\beta}, \gamma+\frac{1}{\gamma}$.

j) Expand $f(x)=x^4-4x^3+8x^2+8x+7$ as a polynomial in x–4.

k) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4-x^3+2x^2+x+1=0$ find the value of $(\alpha+1)(\beta+1)(\gamma+1)(\delta+1)$.

l) Determine the multiple roots of the equation $x^4+2x^3+2x^2+2x+1=0$.

m) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

n) Define reciprocal equation.

o) If m, n are integers prime to each other, then prove that the equations $x^m-1=0$ and $x^n-1=0$ have no common root except 1.

2. Answer any **five** questions: 2×5=10

a) Form the equation whose roots are $\alpha+\beta+\gamma,\ \alpha+\omega\beta+\omega^2\gamma,\ \alpha+\omega^2\beta+\omega\gamma$, where ω is an imaginary cube root of 1.

b) Find the condition that the roots of the equation $x^3-px^2+qx-r=0$ will be in geometric progression.

c) Obtain the condition that $x^3+3px+q$ may have a factor of the form $(x-a)^2$.

d) Find the remainder when $x^5-3x^4+4x^2+x+4$ is divided by (x+1)(x–2).

e) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0,\ (d\neq 0)$, show that $\alpha=-\dfrac{8d}{3c}$.

f) If $\alpha, \beta, \gamma, \delta$ be the roots of the biquadratic equation $x^4+px^3+qx^2+rx+s=0$, find the value of (i) $\Sigma\alpha^2\beta$, (ii) $\Sigma\alpha^2\beta\gamma$.

g) Solve the reciprocal equations $x^4+x^3+2x^2+x+1=0$.

h) Find the special roots of the equation $x^{12}-1=0$.

3. Answer any **two** questions: 5×2=10

a) Solve the equation by Cardan's Method:

$$27x^3+54x^2+198x-73=0$$

b) Solve the equation $x^4-4x^3-4x^2-4x-5=0$, given that two roots α,β are connected by the relation $2\alpha+\beta=3$.

c) If α be an imaginary root of the equation $x^7-1=0$, find the equation whose roots are $\alpha+\alpha^6,\ \alpha^2+\alpha^5,\ \alpha^3+\alpha^4$.

4. Answer any **one** question: 10×1=10

a) i) Find the values of k for which the equation $x^4+4x^3-2x^2-12x+k=0$ has four real and unequal roots. 5

ii) If α, β, γ be the roots of the equation $ax^3+bx^2+cx+d=0,\ d\neq 0$, find the equation whose roots are $\alpha+\dfrac{1}{\alpha},\ \beta+\dfrac{1}{\beta},\ \gamma+\dfrac{1}{\gamma}$. 5

b) i) Solve the equation $x^5-1=0$ and deduce the values of $\cos\dfrac{\pi}{5}$ and $\cos\dfrac{2\pi}{5}$. 5

ii) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are

$\alpha+\beta-2\gamma, \beta+\gamma-2\alpha, \gamma+\alpha-2\beta$.

Deduce the condition that the roots of the given equation may be in arithmetic progression. 5

c) i) Calculate Sturm's function and locate the position of the real roots of the equation $x^3-7x+7=0$. 5

ii) If the equation whose roots are squares of the roots of the cubic $x^3-ax^2+bx-1=0$ is identical with this cube, prove that either a=b=0, or a=b=3, or a, b are the roots of the equation $t^2+t+2=0$. 5

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