U.G. 3rd Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCHT303

Course Title : Geometry-3D and Vector Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) The co-ordinates of the points P, Q, R, S are (3, 4, 5), (4, 6, 3), (–1, 2, 4), and (1, 0, 5) respectively. Find the projection of $\overline{PQ}$ on $\overline{RS}$.

b) Find the values of *l* and a for which the line $\frac{x-2}{l}=\frac{y+3}{4}=\frac{z-6}{-2}$ is perpendicular to the plane $3x-2y+az+10=0$.

c) Find the value of 'a' for which the plane $x+y+z=a$ is a tangent plane to the sphere $x^2+y^2+z^2=27$.

d) Find the equation of the sphere whose extremities of a diameter are (2, 3, 4) and (–1, 0, 5).

e) If a right circular cone has three mutually perpendicular generators then find its semi-vertical angle.

f) Find the radius of the circle given by the equations $x^2+y^2-4x+2y=0$ and z = –1.

g) What type of surface does the equation $x^2+3y^2-2z^2=0$ represent?

h) Find the equation of the tangent plane to the sphere $x^2+y^2+z^2+4x-2y+6z-32=0$ at the point (1, 2, 3).

i) Write the equation of the enveloping cone of $ax^2+by^2=2cz$ with vertex at (α, β, γ).

j) Find the divergence of a vector point function $xz^3\hat{i}-2x^2yz\hat{j}+2yz^4\hat{k}$ at the point (1, –1, 1).

k) Show that if $\vec{a}(t)$ has fixed direction then $\vec{a}\times\frac{d\vec{a}}{dt}=0$.

l) State Green's theorem in a plane.

m) Show that $\vec{\nabla}.\vec{r}=3$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

n) Find the normal derivative of the function $f(xyz) = x^3 - y^2 + 2z^2$ to (1, 2, 3).

o) Find the equation of the line which passes through the point $3\hat{i} - \hat{j} + \hat{k}$ and parallel to the vector $\hat{i} + \hat{j} + \hat{k}$.

2. Answer any **five** questions: 2×5=10

a) Find the condition that the straight lines $\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}$, $\frac{x}{a\alpha} = \frac{y}{b\beta} = \frac{z}{c\gamma}$ and $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ will lie on a plane.

b) Find the nature of the quadratic surface given by the equation

$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$.

c) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$ then evaluate $\int_c \vec{F}.d\vec{r}$ when C is the curve in xy plane given by $y = 2x^2$ from (0, 0) to (1, 2).

d) If for a vector $\vec{r} = \hat{f}(t)$ has a constant magnitude, then prove that $\vec{f} \bullet \frac{d\vec{f}}{dt} = 0$.

e) Determine the points of intersection of the line $\frac{x+2}{-1} = \frac{y+4}{-2} = \frac{z-3}{1}$ and the cylinder $x^2 + z^2 = 1$.

f) Find the equation of the tangent plane to the hyperbolic paraboloid $5x^2 - 2y^2 = 2z$, parallel to the plane $10x - 6y - z = 7$.

g) Find the equation of the right circular cone whose vertex is the origin, axis is the x-axis and semi-vertical angle is 60^o.

h) Find a unit vector parallel to xy-plane and perpendicular to the vector $4\hat{i} - 3\hat{j} + \hat{k}$.

3. Answer any **two** questions: 5×2=10

a) Find the locus of the points of intersection of perpendicular generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$.

b) Reduce the equation

$2x^2 + 20y^2 + 18y^2 - 12yz + 12xy + 22x + 6y - 2z + 2 = 0$

to its canonical form and determine the nature of the quadratic represented by it.

c) Find the directional derivatives of $\phi(x, y, z) = x^2yz + 4xz^2$ at (1, 2, –1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.

4. Answer any **one** question: $10\times1=10$

a) i) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ whose vertex is the point P(x, y, z) by the plane z=0 is a rectangular hyperbola. Show that the locus of the point P is $\frac{x^2+y^2}{a^2+b^2}+\frac{z^2}{c^3}=1$.

ii) Prove that the centre of the spheres which touch the lines y=mx, z=c; y=–mx, z=–c lie upon the conicoid

$mxy+c(1+m^2)z=0$ 6+4

b) i) Find the equation of the tangent plane to the quadratic $\frac{x^2}{2}+\frac{y^2}{3}-\frac{z^2}{4}=1$, which passes through the point (3, 4, –3) and is parallel to the line x = y = –z.

ii) If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$, evaluate $\iint_S(\vec{\nabla}\times\vec{F}).\hat{n}\,dS$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. 6+4

c) i) Verify stoke's theorem (for $\vec{F}=y\hat{i}+z\hat{j}+x\hat{k}$ and S is the upper half surface of the sphere $x^2+y^2+z^2=1$ with bounder Γ).

$$\oint_\Gamma \vec{F}.d\vec{r}=\int_S(\vec{\nabla}\times\vec{F}).\hat{n}dS$$

ii) Using Divergence theorem find $\iint_S \vec{F}.\hat{n}ds$ where

$\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$

and S is the six faces of the rectangular parallelopiped

$0\le x\le a,\ 0\le y\le b,\ 0\le z\le c$.

iii) Show that $\vec{\nabla}.\vec{r}=3$ and $\vec{\nabla}\times\vec{r}=0$.

4+4+2