U.G. 5th Semester Examination - 2021

MATHEMATICS

Course Code: BMTMDSHT1 [DSE1]

Course Title: Linear Programming Problem and Game Theory

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Define a convex set.

b) Determine the convex hull of the points (0,0), (0,1), (1,2), (1,1), (4,0).

c) Examine whether the set $S=\{(x_1,x_2):x_1x_2\le 4\}$ is a convex set or not.

d) Find the dual of the LPP:

Maximixe $Z=3x_1+4x_2$

subject to $x_1+x_2\le 12$

$2x_1+3x_2\le 19$

$x_1\le 8$

$x_2\le 5$

$x_1,x_2\ge 0$

e) Define saddle point and value of a game in game theory.

f) Use dominance property to reduce the following game to a 2 × 2 game.

Player B

Player A			
	1	7	2
	6	2	7
	5	1	6

g) Find the separator of two hyper spaces $x_1=\{x:cx\le z\}$ and $x_2=\{x:cx\ge z\}$.

h) Give an example of a convex polyhedron.

i) Is the solution $x_1=2,\ x_2=4,\ x_3=5$ of the system $2x_1-x_2+2x_3=10;\ x_2+4x_2=18$ a B.F.S. or not?

j) For the objective functions Max Z=CX, we have same values of Z for X = X_1, X_2, X_3, ...X_k. Then find the value of Z for $Y=\sum_{i=1}^{k}\lambda_i X_i$ where $\forall i,\ y_i\ge 0$ and $\sum_{i=1}^{k}\lambda_i=1$.

k) What is the nature of the solution when for particular k, the net evaluation $z_k - c_k < 0$ and corresponding columm elements $y_{ik} \leq 0, \forall i$?

l) When the degenaracy occurs in the simplex table?

m) If some variables of the primal problem be unrestricted in sign, then what is the nature of the corresponding constraint of the dual problem?

n) Write down the following transportation problem into a balanced one:

		To 1	2	3	supply
	1	5	1	7	10
From	2	6	4	6	80
	3	3	2	5	15
Demand		75	20	50	

o) "Assignment problem is a special type of a transportation problem."

2. Answer any **five** questions: 2×5=10

a) Find all the basic feasible solutions of the system of equation

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$

b) Prove that if the objective function assumes its optimal value at more than one extreme point then every convex combination of these extreme points also gives the optimal value of the objective function.

c) Reduce the following LPP fit for first simplex table:

Minimize $z = 2x_1 + 4x_2 + x_3$

Subject to $x_1 + 2x_2 - x_3 \leq 5$
$2x_1 - x_2 + 2x_3 = 2$
$-x_1 + 2x_2 + 2x_3 \geq 1$
$x_1, x_2, x_3 \geq 0$

d) Find the optimal assignment and minimum cost of the following assignment problem:

	I	II	III
A	10	8	12
B	18	6	14
C	6	4	2

e) Show that the 2×2 game $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-strictly determined, if $a < b$, $a < c$, $d < b$ and $d < c$.

f) Explain how degeneracy occur in some middle stage of simplex method.

g) State the fundamental theorem of LPP.

h) Given Initial table to solve LPP by simplex method as

		C_j	2	3	0	0	0
C_B	B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5
0	B_3	2	−1	2	1	0	0
0	B_4	6	1	1	0	1	0
0	B_5	9	1	3	0	0	1
	$z_j - C_j$		−2	−3	0	0	0

write down the problem in canonical form.

3. Answer any **two** questions: 5×2=10

a) Obtain an initial B.F.S. of the following T.P. by VAM:

	D_1	D_2	D_3	D_4	a_i
O_1	10	7	3	6	30
O_2	1	6	8	3	50
O_3	7	4	5	3	70
b_j	30	20	60	40	

b) Solve the following game graphically :

Player B

Player A		B_1	B_2
	A_1	2	−3
	A_2	−2	5
	A_3	0	1

c) For a basic feasible solution $\underset{\sim}{X}_B$ of an LPP Max Z = cx, subject to $Ax = b, x \geq 0$ if $z_j - c_j \geq 0$ for every column $\underset{\sim}{a}_j$ of A, then prove that $\underset{\sim}{X}_B$ is an optimal, solution.

4. Answer any **one** question: 10×1=10

a) i) Examine whether the following set is convex :

$$S = \{(x_1, x_2) : 2x_1 + x_2 \geq 20, x_1 + 2x_2 \leq 80, x_1 + x_2 \leq 50, x_1, x_2 \geq 0\}.$$

Find the extreme points if it is convex. 3+2

ii) $x_1 = 1, x_2 = 0, x_3 = 2, x_4 = 1$ is a feasible solution to the set of equations

$$2x_1 + 3x_2 + 3x_3 - x_4 = 7$$
$$x_1 + 5x_2 + 2x_3 + x_4 = 6$$

Reduce the feasible solution to all possible basic feasible solutions. 3+2

b) i) Show that there is an unbounded solution to the following LPP:

$$\text{Max } Z = 4x_1 + x_2 + 4x_3 + 5x_4$$

$$\text{S.T. } 4x_1 - 6x_2 - 5x_3 + 4x_4 \geq -20$$

$$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$

$$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$

$$x_1, x_2, x_3, x_4 \geq 0$$

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ii) Solve the following LPP graphically:

$$\text{Max } Z = -x_1 + 2x_2$$

$$\text{S.T. } -x_1 + x_2 \leq 1$$ 5

$$-x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

c) i) Solve the TSP:

	A	B	C	D	E
A	∞	2	4	7	1
B	5	∞	2	8	2
C	7	6	∞	4	6
D	10	3	5	∞	4
E	1	2	2	8	∞

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ii) Transform to LPP and hence solve the game problem whose pay-off matrix is

A \ B			
	2	−3	4
	−3	4	−5
	4	−5	6

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