U.G. 6th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMDSHT6

Course Title : Point Set Topology

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) State Axiom of choice.

b) What do you mean by well ordered set?

c) Define the indiscrete topology on a set X.

d) Determine the accumulation points of the set $(a, b] \subseteq \mathbb{R}$.

e) Let X be a discrete topological space. Determine the closure of any subset A of X.

f) Find the closed sets for the topological space (X, τ) where $X=\{a, b\}$, $\tau=\{\phi, \{a\}, X\}$.

g) The open discs form a base for the usual topology on the plane $\mathbb{R}^2$. Is it true?

h) Consider the topology on $X=\{a, b, c, d, e\}$ as topology

$$\tau=\{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, c\}\}.$$

List the neighbourhoods of the point e.

i) Let the real function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x)=x^2$. Show that f is not open.

j) Totally bounded sets are bounded. Is it true?

k) $\mathbb{R}^2 \setminus \{(0, 0)\}$ is a disconnected set. Is it true?

l) State true or false of the following statement "$\mathbb{R}$ is a compact set".

m) Let $f: A \to \mathbb{R}$, $(A \subseteq \mathbb{R})$ be a continuous function and A is a connected subset of $\mathbb{R}$. State whether the image f(A) is connected or not.

n) $[a, \infty)$ is a compact subset of $\mathbb{R}$. Is it true?

o) Define a perfect set.

2. Answer any **five** questions: 2×5=10

a) Determine whether $d(x, y)=|x-2y|$, $x, y \in \mathbb{R}$ is a metric on $\mathbb{R}$.

b) Determine interior and closure of Q in $\mathbb{R}$ with usual metric.

c) Find the limit points of any subset A of a discrete metric space.

d) Find the closed subsets for the topology τ on $X=\{a, b, c, d, e\}$ where

$$\tau=\{\phi, X, \{a\}, \{c, d\}, \{b, c, d, e\}, \{a, c, d\}\}.$$

e) Let τ be the class consisting of $\mathbb{R}$, ϕ and all infinite open intervals $A_q=(q, \infty)$ with $q \in Q$, the rationals. Show that τ is not a topology on $\mathbb{R}$.

f) List all topologies on X={a, b, c} which consist of exactly four members.

g) Show that $\mathbb{R}$ is homeomorphic to (0, 1) w.r. to usual topology.

h) Prove or disprove: $\mathbb{R}$ with co-finite topology is Hausdorff.

3. Answer any **two** questions: 5×2=10

a) Prove that the unit interval [0, 1] is non-denumerable.

b) Let $f: X \to Y$ be a function from a non-empty set X into a topological space (Y, μ). Furthermore, let τ be the class of inverses of open subsets of Y:

$$\tau=\{f^{-1}[G]: G \in \mu\}.$$

Show that τ is a topology on X.

c) Prove that continuous images of a compact set is compact.

4. Answer any **one** question: 10×1=10

a) i) Let A be a subset of the topological space X. Let A' be the set of limit points of A. Then prove that $\bar{A}=A \cup A'$.

ii) Show that a subspace of a Hausdorff space is Hausdorff. 5+5

b) i) Let X={a, b, c, d, e} and let $\mathcal{A}$={{a, b, c}, {c, d}, {d, e}}. Find the topology on X generated by $\mathcal{A}$. 4

ii) Show that all intervals (a, 1] and [0, b) where 0<a, b<1 form a subbase for the relative usual topology on the unit interval I=[0, 1]. 4

iii) Show that every discrete space X is locally connected. 2

c) i) Let $f: X \to \mathbb{R}$ be a real continuous function defined on a connected set X. Then f assumes as a value each number between any two of its values. 4

ii) Prove that every complete metric space X is of second category. 6

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