U.G. 1st Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCHT101

Course Title : Calculus & Analytical Geometry (2D)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $1\times10=10$

a) State L'Hospital's rule.

b) If $y=e^{\tan^{-1}x}$, then show that $(1+x^2)y_2+(2x-1)y_1=0$, where y_n is the n-th derivative of y w.r.to x.

c) If the origin be shifted to the point (2, –1) without changing the direction of axes, find the form of the equation 2x–3y=8 in new co-ordinate system.

d) Find the length of the cartesian subnormal for the parabola $y^2=4ax$.

e) Find the point on the conic $\frac{5}{r}=1+2\cos\theta$ whose vectorial angle is $\frac{\pi}{3}$.

f) Prove that the curve $y=\log x$, is concave w.r.to x-axis if $x>1$ and is convex if $0<x<1$.

g) Write down the formula for the radius of curvature of the parametric equation x=f(t), $y=\phi(t)$.

h) Find the value of $D^n(ax+b)^n$.

i) What will be the equation of the normal to the ellipse $\frac{x^2}{16}+\frac{y^2}{12}=1$ at the point (2, 3)?

j) On the conic $r=\frac{21}{5-2\cos\theta}$, find the point with the least radius vector.

k) Determine the nature of the conic represented by the equation $x^2-2xy+2y^2-4x-6y+3=0$.

l) Give an example of non-singular or non-degenerate curve.

m) Find the area of the circle $r=2a\sin\theta$.

n) Define asymptote of a curve.

o) If $I_n = \int x^h e^{-x} dx$, then show that $I_n = -e^{-x}x^n + nI_{n-1}$.

2. Answer any **five** questions from the following: $2\times5=10$

a) Find the area of the region bounded by the curves $y=x^2$ and $x=y^2$.

b) If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

c) Find the pedal equation of the cardioide $r = a(1+\cos\theta)$.

d) Show that the radius vector is inclined at a constant angle to the tangent at any point on the equiangular spiral $r = ae^{b\theta}$.

e) Find the points of inflexion of the curve $y = (\log x)^3$.

f) If $z = (x+y)\phi\left(\frac{y}{x}\right)$, where ϕ is an arbitrary function, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$.

g) Find the reduction formula for $\int \cos^n x \, dx$.

h) Find the volume of the solid generated by the rotation of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its major axis.

3. Answer any **two** questions: $5\times2=10$

a) Show that the equation $8x^2 + 10xy + 3y^2 + 22x + 14y + 15 = 0$ represents a pair of intersecting straight lines. Find their point of intersection and the angle between them. $3+(1+1)=5$

b) Show that the area of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx + my = 1$, is $\frac{\sqrt{h^2-ab}}{am^2 - 2hlm - bl^2}$.

c) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1+\cos\theta)$ about its base. 2+3

4. Answer any **one** question: $10\times1=10$

a) i) State and prove Euler's theorem for a homogeneous function in two variables.

ii) Show that the envelope of the family of lines $y = mx + \frac{a}{m}$; a is fixed and m is a variable parameter, is a parabola.

iii) Find all the asymptotes of $xy^2 - y^2 - x^3 = 0$.

4+2+4

b) i) Show that the conics $\frac{l_1}{r} = 1 - e_1 \cos\theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ will touch each other if

$$l_1^2(1-e_2^2) + l_2^2(1-e_1^2) = 2l_1 l_2 (1 - e_1 e_2 \cos\alpha).$$

ii) Determine $\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\frac{\pi x}{2a}}$.

iii) If $u = e^{xyz}$, show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2).e^{xyz}.$$

4+2+4

c) i) If $y = \sin(m \sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

ii) If $J_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then show that

$$J_{n+1} + J_{n-1} = \frac{1}{n}.$$

iii) Find the point of inflexion of the curve $y^2 = x(x+1)^2$.

4+4+2
