U.G. 5th Semester Examination - 2020

MATHEMATICS

Course Code: BMTMDSHT2 [DSE 2]

Course Title: Mechanics-1

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

   a) Define potential energy of a system of particles.

   b) How does a rigid body differ from a deformable body?

   c) Is spherical polar co-ordinate system an inertial frame? Justify.

   d) Define angular momentum of a system of particles.

   e) Write down the Gallilean Transformations.

   f) Give an example of non-inertial frame.

   g) When a reference frame is said to be an inertial reference frame?

   h) State principle of virtual work.

   i) Write down the vector equation of motion of the centre of mass of a moving body.

   j) Define centre of mass of a system of particles.

   k) Write the inertia matrix with respect to the principal axes.

   l) State D'Alembert's Principle.

   m) What do you mean by 'constraints on a system'?

   n) What is the necessary and sufficient conditions for a force $\vec{F}$ to be conservative force?

   o) State the theorem of Perpendicular Axis.

2. Answer any **five** questions: 2×5=10

   a) A moving system consists of three particles of masses 2, 3, 4 units located at (1, 0, 1), (0, –1, 0), (2, –2, –2). Find the kinetic energy of its centre of mass.

   b) Obtain moment of inertia of a circular plate about a line in its own plane whose distance from the centre is *d*.

c) What are principal axes? What is the form of inertia matrix with respect to principal axes?

d) Distinguish between internal and external forces as explained by Newton's laws of motion.

e) State which of the following forces can be termed as 'action-at-a-distance':

i) gravitational force;

ii) magnetic force;

iii) elastic force;

iv) viscous force.

f) Establish the relation between the rate of change of angular momentum of a moving particle and the force acting on it.

g) Find the work done by a force on a particle for a given time interval in terms of the change of their kinetic energy.

h) A system of n particles of masses $m_i(i=1, 2, .., n)$ moves under external forces and mutual actions and reactions. Write the equation of motion of the i-th particle and deduce the equation of motion of the centre of mass.

3. Answer any **two** questions: 5×2=10

a) Obtain the equation of motion of a plane lamina rotating about a fixed axis perpendicular to the plane of the lamina in the form $MK^2\frac{d^2\theta}{dt^2}=L$, the symbols are to be explained by you. 5

b) A uniform rod OA of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about the downward vertical OZ. Using D'Alembert's principle or otherwise, show that the inclination of the rod to the vertical is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. 5

c) Find the moments and products of inertia with respect to rectangular axes, parallel to the co-ordinate axes through the point $(a, 0)$ on the circumference for the uniform circular disc $x^2+y^2=a^2$ of mass M. Also write down the corresponding inertia matrix. 5

4. Answer any **one** question: 10×1=10

a) i) Obtain the inertia matrix for a homogeneous rectangular plate of mass M bounded by $x=\pm a$, $y=\pm b$ with respect to the co-ordinate axes.

ii) The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$ and $2b$. Show that the inclination to AB of one of the principal axes at A is $\frac{1}{2}\tan^{-1}\frac{3ab}{2(a^2-b^2)}$.

iii) A circular hoop of radius 'a' rolls down a perfectly rough inclined plane of inclination α to the horizontal. Find the acceleration of the hoop down the plane.

4+4+2

b) i) Discuss briefly the Gallilean Transformation and show that the form of the Newton's second law of motion remains invariant under such transformation.

ii) A homogeneous sphere of radius 'a' rotating with angular velocity 'ω' about a horizontal diameter is gently placed on a table whose co-efficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2a\omega}{7\mu g}$ and that the sphere will roll with angular velocity $\left(\frac{2\omega}{7}\right)$.

4+6

c) i) A uniform solid cylinder rolls, with generators horizontal, down an inclined plane of inclination α. Prove that the condition for pure rolling is that the co-efficient of friction must be greater than equal to $\frac{1}{3}\tan\alpha$.

ii) A uniform rod of length $2a$ is placed with one end in contact with a horizontal table and is at an inclination α to the horizon, and is allowed to fall when it becomes horizontal. Show that its angular velocity is $\sqrt{\frac{3g}{2a}\sin\alpha}$, whether the plane be perfectly smooth or perfectly rough. Show also that the end of the rod will not leave the plane in either case.

5+5

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