U.G. 5th Semester Examination - 2020

MATHEMATICS

Course Code: BMTMDSHT1 [DSE1]

Course Title: Linear Programming Problem and Game Theory

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) What is the convex hull of the set

$$S=\left\{(x,\ y): \frac{x^2}{3}+\frac{y^2}{2}=1\right\}?$$

b) Express (5, 2, 1) as a linear combination of (1, 1, 0) and (3, 0, 1).

c) State the condition for unbounded solution of an LPP in simplex algorithm.

d) Check whether the solution $x_1=2$, $x_2=0$ and $x_3=1$ is a basic solution or not to the following set of equations

$$2x_1+x_2-x_3=3$$

$$x_1+2x_2+3x_3=5$$

e) Determine the position of the point $X=(1,\ 2,\ 3,\ -5)$ relative to the hyperplane $2x_1+3x_2+4x_3+5x_3=7$.

f) What is the necessary and sufficient condition for a point $X \geq 0$ in the convex set S of all feasible solutions of the system AX=b, $X \geq 0$ to be an extreme point?

g) Write down the standard primal problem and its dual problem in matrix form.

h) Find the dual of the LPP:

Minimize $z=3x_1+4x_2$

subject to $2x_1+4x_2 \geq 60$

$3x_1+2x_2 \geq 50$

$x_1,\ x_2 \geq 0$

i) In Hungarian method if the number of lines drawn to cover the zeros is less than the order of the cost matrix than what is done?

j) Prove that $x_{ij} = \frac{a_i b_j}{M}$; $[i=1, 2, ..., m;\ j=1, 2, ..., n]$,

where $M = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is a feasible solution of a transportation problem.

k) Define a loop in a transportation table.

l) Write down the total number of possible solutions for n×n assignment problem.

m) Define artificial variable in Linear Programming Problem.

n) Examine whether the set $S = \{(x_1, x_2) : x_1^2 + x_2^2 = 4\}$ is a convex set or not.

o) Use dominance property to find the value of the game:

	B_1	B_2
A_1	5	4
A_2	10	25

2. Answer any **five** questions: 2×5=10

a) $x_1=1$, $x_2=3$, $x_3=2$ is a feasible solution of the equations:

$$2x_1+4x_2-2x_3=10$$
$$10x_1+3x_2+7x_3=33$$

Reduce it to a basic feasible solution.

b) Find the feasible region bounded by the constraints:

$$x_1 + x_2 \geq 2$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 - x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

c) Using north-west corner rule find the initial basic feasible solution to the following transportation problem:

	D_1	D_2	D_3	Availability
O_1	3	8	7	10
O_2	6	5	8	5
Demands	6	5	4	

d) Show that a hyperplane is always a convex set.

e) For the game with pay-off matrix:

		Player A		
		I	II	III
Player B	I	−1	2	−2
	II	6	4	−6

Determine the best strategies for player A and B and also the value of the game for them.

f) Find two basic feasible solutions of the system

$$x_1+2x_3=1$$

$$x_2+x_3=4$$

g) Reduce the following LPP to its standard form:

Maximize $z=2x_1-x_2+2x_3$

subject to $x_1+x_2-3x_3 \le 8$

$4x_1-x_2+x_3 \ge 2$

$2x_1+3x_2-x_3 \ge 4$

$x_1, x_2, x_3 \ge 0$

h) Formulate mathematically an assignment problem.

3. Answer any **two** questions: $5\times2=10$

a) Prove that the objective function of a linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solution.

b) Solve the following LPP by simplex method:

Maximize $z=5x_1+3x_2$

subject to $3x_1+5x_2 \le 15$

$5x_1+2x_2 \le 10$

$x_1, x_2 \ge 0$

c) Solve the following game problem by algebraic method:

B

	−2	3	−1
A	4	−1	2
	1	2	3

4. Answer any **one** question: $10\times1=10$

a) i) If for a basic feasible solution X_B of a linear programming problem

Maximize $Z=CX$

subject to $AX=b,\ X\ge0$

We have $Z_j-C_j \ge$ for every column a_j of A, then prove that X_B is an optimal solution.

ii) Solve graphically:

Maximize $Z=9x+8y$

subject to $4x+3y \le 30$

$2x+3y=18$

$x, y \ge 0$ 7+3

b) i) Using dual simplex method, show that the following LPP has no feasible solution:

Minimize $Z=x_1+x_2$

subject to $2x_1+x_2 \geq 2$

$-x_1-x_2 \geq 1$

$x_1,\ x_2 \geq 0$

ii) Obtain an optimal BFS for the following transportation problem: 4+6

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

c) i) Solve the following game graphically:

Player B

Player A			
	3	−3	4
	−1	1	−3

ii) Solve the assignment problem where the assignment cost of assigning any operation to any one machine is given below:

Operators

Machines		A	B	C	D
	I	1	4	6	3
	II	9	7	10	9
	III	4	5	11	7
	IV	8	7	8	5

5+5

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